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A new symbolic-based continuous (infinite) modal approach for systems control and operation using computational mathematics

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ABSTRACT

This paper presents the new concept of symbolic-based continuous (infinite) modal approach for systems control and operation in varying parameters environment. In this approach, the system is run under varying parameters form where the variations are sensed and fed back in a continuous manner for determining using symbolic-based expressions the corresponding control functions. The approach permits the simultaneous manipulation of system control and operation with the online changes of system parameters. The generic symbolic approach enables high system operation flexibility, while the continuous (infinite) modal operation ensures complete smoothing system behavior and full operation modal compatibility all over various system varying parameters regions. The realization of the continuous (infinite) modal design is carried out through symbolic-based embedded control expressions and using computational mathematics. The **proof of concept** of the presented scheme is demonstrated through both an illustrative example and an application representing the implementation of control of inverted pendulum system with varying cart mass operation. It is successfully elucidated that the exact generic tracking of changing parameters could effectively be achieved in very smoothly and with complete continuous modal compatibility and full auto or self-modality. Moreover, the proposed scheme enables providing more performance flexibility compared to the known unimodal and multimodal operation approaches and adds a new dimension for future system control and operation specially in the fields of mechatronics and robotics where smooth and flexible system behavior are to be maintained all over the modes of system operation. Finally, the applications of the suggested approach to examples of parameters varying direct and networked cyber-physical systems are addressed such that the varying parameters are monitored online and continuously fed to the symbolic-control function to yield at the end the exact/generic flexible control scheme.

1. Introduction

Real life systems of varying parameters may have many operation modes (**modal control**) depending on the ranges of such varying parameters. Each mode could be identified and controlled with a particular assigned eigenvalues stabilization values. This approach, however, should include mode switching that could cause sudden system effects or to the occurrences of some sort of incompatibilities. In this respect, "**modality**" is the way or mode in which something exists or done[1]. Modal control is in general an effective approach and could be grouped with the active control techniques [1].

Previous work in the literature of modal synthesis of state space systems was conventionally carried out using eigenvalue/eigenvector analysis [2,3]. It is usually linked with the individual fundamental physical behavioral components of a time response. Modal analysis could allow the breakdown of the responses of a

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[1] The word ***modal*** or ***modality*** is linguistically defined in general as something relating to mode, manner or form. It contains provision as to the mode of procedure or manner taken effect.

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multivariable system into several finite individual modes with some physical meanings. Modal analysis has commenced with mechanical systems with structural vibration control, where mode shapes are arrived through from various measurements and frequency analysis. The concept was then extended after that to general classes of mechanical and non-mechanical systems exhibiting modal behaviour in their responses [4].

There are ample numbers of research work addressing the implementation of modal analysis since the late sixties of the last century in linear, nonlinear and large-scale control systems design such as given in references [5,6]. These approaches applied numeric-based techniques for the realization of the control functions with pre-specified distinct (or multiple) and real (or complex) eigenvalues assignment for each mode space of operation of the linear or linearized systems. The specification of the designed system is that some or all system eigenvalues are moved to certain desired locations and usually assumed that the system is invariant for each mode of its operation. For multimodal systems, a switching process has to be carried out for assigning the pre-specified poles at each corresponding mode.

2. Work motivation

The main problem of discrete, switching or multimodal approach for systems control and operation is to determine the state of the models (process, controlled process and specification), when a mode of the parameters varying system has to leave the initial mode to convert to another one, called the final mode. This aspect will be referred to as the compatibility problem. In fact, the conversion is possible if the initial mode is in a compatible state with the final mode. Compatible means the state of the shared components between initial and final mode are the same. It also means that the existing requirements in both modes should still be respected even if the system switches its modes.

The above compatibility problem of the multimodal approaches for parameters varying control systems [7–9] gives rise to the importance of looking for all through smooth and compatible modal approach. This will be achieved in this work through introducing the continuous (infinite) modal approach for active control system design.

The new approach is developed based on symbolic-based representation of system control. In this case, operation solution is obtained in a form of generic expressions derived by the assistance of computational mathematics.

3. Background on parameters varying system control and operation

The ***parameters varying systems*** (PVS) are defined as the type of systems whatsoever their type (linear or nonlinear, dynamic or static, ...) where parts or all of their parameters or systems control and operation coefficients are not *primarily* constant and could vary due to some causes. This could lead to overall system changes due to some ***natural*** or ***man-made*** causes [10,11]. Such parameter varying change will be denoted in this work by the symbol "ρ".

A control system is referred to as **Parameters Varying System** (PVS) when its parameters are dependent on a varying parameter say "ρ". These parameters variations could be fast varying or slow varying types. The types of PVS parameters variation could be classified as chaotic, fuzzy, probabilistic or random, periodic or others[2].

[2] In this research, the analysis will be mainly based on parameters varying systems with variation expressed by exact explicit mathematical (algebraic) expressions, excluding cases of probabilistic or random, fuzzy and chaotic behavior

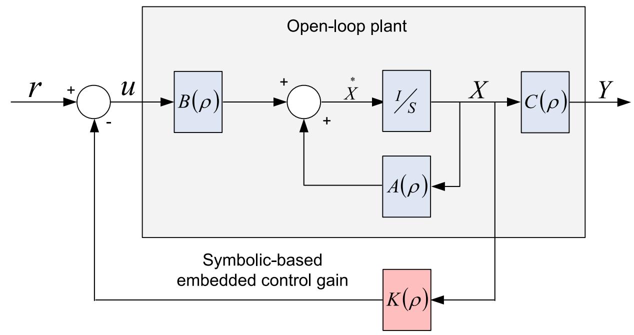


Fig. 1. A typical example of symbolic-based embedded feedback control system.

A typical parameter varying control system is shown in Fig. 1 and can be expressed in the state-space form as a function of varying parameters say "ρ" as follows:

$$\begin{aligned}\dot{x} &= A(\rho).x(t) + B(\rho).u(t), t \geq 0, x(0) = x_0 \\ &\text{and} \\ z(t) &= C(\rho).x(t) + D(\rho).u(t)\end{aligned} \tag{1}$$

where x, u and z are respectively the state, the input and the output of the system. These states could be in general of the multi-dimensional form. The parameters varying vector ρ acts internally on the system by modifying its structure, and consequently altering the overall input-output behavior of the system. It is assumed usually that the above matrices are continuous and bounded functions.

The corresponding general nonlinear PVS representation of (1) can be represented as:

$$\dot{x} = f[x(t), \rho, \dot{\rho}, u(t)], t \geq 0, x(0) = x_0 \text{ and } u(0) = u_0 \tag{2}$$

such that $f[.]$ is a general nonlinear function of its arguments.

The parameter varying control system could be classified according to its behavior to three classes as follows: Discrete behavior, Continuous behavior and Discontinuous behavior. In this work, emphasis will be based on the continuous behavior type.

4. Modal systems control and operation framework

Several definitions of modality can be found in the relevant literature. These can be broadly categorized into three general groups: Physiological/human-centered, technology/system-centered, and definitions incorporating both views. In this work, we will adopt the second definition [12].

In dealing with the realization of system control and operation schemes, we could have three alternatives: (i) Single (one) **unimodal** (ii) Discrete, switching (finite) or **multimodal**, or (iii) **Continuous** (infinite) modals, as elucidated in Fig. 2 and described hereafter.

4.1. Single (one) or unimodal operation

In this type of operation, the assigned designs are kept the same despite change of the system varying parameters and the system control and operation parameters are calculated in their symbolic form accordingly. The control operation has only single modal operation regardless of any changes of system parameters.

4.2. Discrete, switching (finite) or multimodal operation

In this type of operation, the system is characterized by having multiple modes depending on the range of the varying parameters [6]. Each mode of operation has its own assigned design poles that

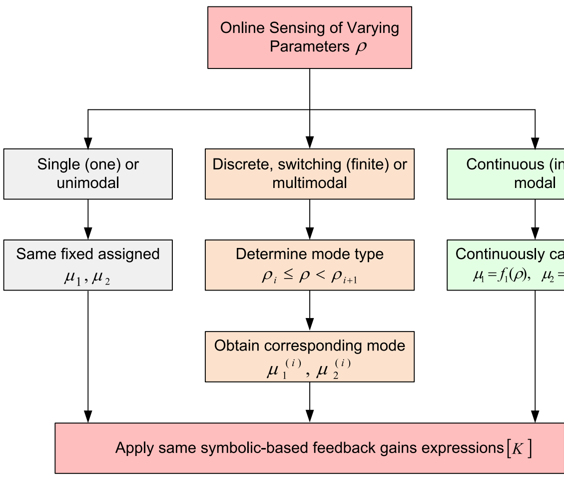


Fig. 2. Various classifications of modal systems control and operation schemes.

are accordingly applied. The modal compatibility problem, however, could arise when transferring from one mode to another.

4.3. Continuous (infinite) modal $\boldsymbol{M}^{\infty}$ operation (proposed)

In this type of operation, continuous modal operation of the system is carried out by continuously determining based on the sensed varying parameters the assigned designed system control and operation schemes using relevant appropriate continuous function of such parameters changes. The transfer is made in a smooth manner with full continuous (infinite) modal compatibility all over the operation regions. This type of modal operation will be denoted by $\boldsymbol{M}^{\infty}$ where the superscript denotes infinite number of modal operations.

In this symbolic-based representation approach all the unimodal, multimodal, and continuous modal operations are implemented at the end using the same schemes expression formula. This gives rise to the necessity of following the symbolic-based approach in the derivation of such expressions of such system control and operation schemes. Such expressions could then be regarded as exact generic solutions of such schemes.

While the current multimodal approach is assuming finite number (discrete) of system modes and switches from one to another using different assigned schemes at each mode, the continuous (infinite) modal form do not perform any switches but maintain smooth continuous operation thru continual symbolic-based scheme changes with the system varying parameters. Therefore, the proposed technique is equivalent of having infinite number of modes. The new technique provides smooth operation of the system behaviour and avoids the previous non-compatibility reactions of multimodal approaches when shifting from one mode to another.

5. Symbolic-based continuous (infinite) modal $\boldsymbol{M}$ system control and operation framework

5.1. Derivation of symbolic-based continuous (infinite) modal $\boldsymbol{M}^{\infty}$ control system framework

The derivation of symbolic-based control strategy follows the same mathematical steps that could be performed manually by hand for small-order systems. The derivation of the problem solution could also be assisted by symbolic-based software for higher and more complex systems using pole placement (assignment) approach [13–15].

Let us consider as a demonstration of such derivation, the formulations for the second-order parameters varying system expressed as:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}, and\ C = [1 \quad 0]. \tag{3}$$

Eq. (3) has the same structure of (1), but the varying parameters index was omitted for simplicity.

The pole placement (assignment) approach [16,17] is selected for this illustrative example for deriving the symbolic-based control functions expressions, such as:

$$|sI - A + BK| = 0 \tag{4}$$

where

$$u = -K.x \tag{5}$$

Thus, the modified state-space equation can be written as:

$$\begin{gathered} \dot{x} = (A - B.K).x = \tilde{A}.x \\ \text{where } \tilde{A} = A - B.K. \end{gathered} \tag{6}$$

If we select the desired poles as μ_1 and μ_2 , then we have:

$$|sI - A + B.K| = \left|sI - \tilde{A}\right| = (s - \mu_1)(s - \mu_2) = s^2 + \alpha_1.s + \alpha_2 = 0. \tag{7}$$

Since the Cayley-Hamilton theorem states that $\tilde{A}$ satisfies its own characteristic equation, then we have [16]:

$$\Phi\left(A\right) = \tilde{A}^2 + \alpha_1.\tilde{A} + \alpha_2.I = 0. \tag{8}$$

Moreover, the system controllability matrix CM could be obtained by following formula:

$$C.M = [B \quad A.B] \tag{9}$$

Because this system is controllable the inverse of CM exists, and is given by

$$[B \quad B.A]^{-1}.\Phi(A) = \begin{bmatrix} \alpha_1.K + K.A \\ K \end{bmatrix}. \tag{10}$$

Pre-multiplying both sides of (13) by $[0 \quad 1]$, yields:

$$K = [0 \quad 1] * C.M^{-1} * \Phi(A) = [k_1 \quad k_2]. \tag{11}$$

Solving (10) and (11) for the symbolic-based control gains k_1 and k_2 and simplifying, gives the following expressions for the system control functions:

$$k_1 = \frac{b_{11}.(a_{11}.a_{21} + a_{21}.a_{22} + a_{21}.\mu_1.\mu_2) - b_{21}.(a_{11}^2 + a_{11}.\mu_1.\mu_2 - \mu_1 - \mu_2 + a_{12}.a_{21})}{a_{21}.b_{11}^2 - a_{12}.b_{21}^2 - a_{11}.b_{11}.b_{21} + a_{22}.b_{11}.b_{21}} \tag{12}$$

and

$$k_2 = \frac{b_{11}.(a_{22}^2 + a_{22}.\mu_1.\mu_2 - \mu_1 - \mu_2 + a_{12}.a_{21}) - b_{21}.(a_{11}.a_{21} + a_{21}.a_{22} + a_{12}.\mu_1.\mu_2)}{a_{21}.b_{11}^2 - a_{12}.b_{21}^2 - a_{11}.b_{11}.b_{21} + a_{22}.b_{11}.b_{21}} \tag{13}$$

Consider now the following illustrative example:

$$A = \begin{bmatrix} 0 & 1 \\ \rho & \rho \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, and\ C = [1 \quad 0] \tag{14}$$

where "ρ" is a varying parameter.

Substituting for the illustrative example parameters in both (12) and (13), we can obtain after simplification the corresponding symbolic-based control functions:

$$k_1 = \rho + \mu_1 + \mu_2 \tag{15}$$

and

$$k_2 = \rho^2 + \mu_1.\mu_2 \tag{16}$$

Applying the continuous modal operation in a generic form for the PVS, we introduce the following general formulas for determining $\mu_1 and\ \mu_2$ using relevant appropriate continuous function of such parameters changes as follows:

$$\mu_1 = f_1(\rho, \dot{\rho}, \ddot{\rho}, ...) \tag{17}$$

and

$$\mu_2 = f_2(\rho, \dot{\rho}, \ddot{\rho}, ...) \tag{18}$$

Table 1
List of some selected mathematical class types in systems control and operation with description of operations within each class [17].

Ser.	Class Type	Description of operations within Class
1.	**Algebraic (General)**	Solving algebraic equations, differential equations, systems of DEs, nonlinear equations, non-polynomial eqs., inequalities, Inverse functions, .. etc.
2.	**Complex Variables**	Handling complex variables, complex functions, complex conjugate, operations (+, −, *), polar forms of complex var., functions of complex var., .. etc.
3.	**Differentiations**	Performing single derivatives $d(.)/dx$, double deriv. $d^2(.)/dxdy$, multiple derivatives, partial derivatives $\partial(.)/\partial x$, double partial derivatives, higher-orders partial deriv., gradient vector, maxima and minima, ..etc
4.	**Functions**	These contain algebraic functions, trigonometric functions, exponential functions, polynomial functions, piecewise functions, composite functions, special functions, Taylor series expansion, Fourier series expansion, two-variable functions, multi-variate functions, summation of functions, periodic functions, harmonic functions, discrete data functions, digital data function, limits of expressions, sequences, ...etc.
5.	**Integrations**	These compromise single integration $\int(.)dx$, double integ. $\iint(.)dx.dy$, multiple integration. contour integration, integral functions, line integrals, surface integrals, improper integrals, .. etc.
6.	**Matrices**	These include matrix operations (+,−,*), matrix transpose (.)^T, matrix inversion (.)^(−1), matrix trace, trace(.), determinant, rank of matrices, norm of matrices, Jacobian matrices, functions of matrices, Jordan canonical form, decomposition of matrices, ..etc.
7.	**Preparations**	These compromise parameters preparations, vectors preparations, matrices preparations, equations preparations, inequalities preparations, ..etc.
8.	**Factorizations**	These include factorizations of polynomials, Eigen-values of matrices, Partial fractions roots of functions, poles of expressions, zeros of expressions, ..etc.
9.	**Simplifications**	The lists contain simplifications of Simpl. functions expressions, matrix operations, complex expressions, symbolic summation, discrete data expressions, digital data expressions, s-transform express., F-transform expressions, other transforms expres., algebraic equations, nonlinear equations, multidimensional syst., ..etc.
10.	**Transformations**	These compromise transformations of S-Transform $\int(.)$, F-Transform $\int(.)$, z-Transform $\sum(.)$, other transforms, Inverse of s-transform express., Inverse of F-transform express., Inverse of z-transform express., Inverse of other transf. express., Trans. multiple integrals, ..etc.

and $f_1(.)\ and\ f_2(.)$ are general continuous functions of their arguments.

For simplicity of investigation, we will consider for this illustrative example the case of slowly varying parameters "ρ", such that $\mu_1 = f_1(\rho)$ and $\mu_2 = f_2(\rho)$. Implementation to the general fast varying parameters following (17) and (18) will follow similar procedures as the proposed approach is based on exact generic symbolic platform,

It follows then from above that the symbolic-based control gains expressions of the continuous (infinite) modal operation can be expressed respectively as:

$$k_1 = \rho + f_1(\rho) + f_2(\rho) \tag{19}$$

and

$$k_2 = \rho^2 + f_1(\rho).f_2(\rho). \tag{20}$$

These symbolic-based control gains expressions can be realized by continuously monitoring the system varying parameters "ρ" thru embedded control configuration and using Functional Programming software facility.

In fact, the most challenging aspect in the continuous (infinite) modal control is the most appropriate selection of the functions $f_1(.)\ and\ f_2(.)$ that map the PVS changes to corresponding system assigned poles in a manner that maintains the best performance of the system behavior all over its various operations regions.

5.2. The strength of symbolic-based system control and operation framework

In general, the system control and operation design involve a very wide range of simple, moderate and sophisticated approaches each for certain type of continuous, discrete or digital data systems. Examples of these design techniques are PID controllers, pole assignment, adaptive control, robust control, quadratic regulators and observers, etc [10].

The basic symbolic-based operation of control systems analysis and design could briefly be classified as follows (see Table 1 for additional details) [17]:

i) **Algebraic (general):** solving algebraic equations of various types.
ii) **Complex variables**: handling complex variables and functions
iii) **Differentiations**: performing different types of ordinary and partial derivatives.
iv) **Functions:** include all types of algebraic functions and expressions.
v) **Integrations:** comprise all different types of single and multiple integrations.
vi) **Matrices:** include all matrix operations and manipulations.
vii) **Preparations:** comprise parameters and equations preparations.
viii) **Factorizations:** include factorization of polynomials and expressions,
ix) **Simplifications:** include simplifications of functions, expressions, and equations.
x) **Transformations:** comprisehandling all transforms and the inverse of these transforms.

There are many advantages for symbolic representation, in contrary to numerical computations. The computer algebra- also called algebraic computation or symbolic computation- use symbols in computations and representing mathematical objects, such as an equation, a function, a group, or any other mathematical form. Modern computer algebra systems provide a powerful programming language. Matlab Symbolic Toolbox with MuPAD, Mathematica, Idris, and Python are some examples of such computer algebra packages [18,19].

6. Realization of symbolic-based system control and operation using embedded configuration and computational mathematics

The incorporation of system control and operation expressions in a symbolic (mathematical) form is a new recent trend matching parameter varying nature of the problem. This enables addressing the problem in an exact generic manner, more than any other numeric approaches. The realization of such symbolic expressions is carried out through computational mathematics. The system is based on the embedded control configuration and using **Functional Programming** as shown in Fig. 3**.** In this figure, the control expressions are implemented as a function of the parameter varying index "ρ".

An **embedded system** is a computer system with a dedicated function within a larger mechanical or electrical system, often with real-time computing constraints [18–20]. It is embedded as part of a complete device often including hardware and mechanical parts. Exampled of these embedded systems are single 8- , 16-, or 32-bit microcontroller based on complex instruction set computer (CISC) architecture, application-specific integrated circuits (ASICs) and/or field-programmable gate arrays (FPGAs) [21].

Functional programming is basically a style of computational mathematics which models computations as the evaluation of expressions. In functional programming, programs are executed by evaluating expressions, in contrast with imperative programming where programs are composed of statements which change global state when executed. Functional programming typically avoids using mutable state. Examples of software that supports functional programming are Common Lisp, Wolfram Language (known as Mathematica), Haskell, Erlang, F#, and many others [22].

In general, the combination of incorporating embedded configuration systems with Functional Programming will provide many benefits such as handling complex symbolic functions/expressions, real time and concurrent computations problems, , multiple rate problems, performing programming without side-effects; all in exact, transparent and strong determinacy manner [23,24].

7. Implementation of proposed continuous (infinite) modal system control and operation framework to a case study

The mathematical (symbolic-based) **continuous (infinite) modal** procedure for the case study of parameters varying inverted pendulum control will follow two basic steps, namely: i) **Derivation** of the continuous (infinite) modal symbolic-based control functions and ii**) Implementation and experimentation** of derived symbolic-based control expressions thru embedded microcontroller configurable system using functional programming.

7.1. Derivation of continuous (infinite) modal system control and operation expressions

The case study represents the control and operation of the continuous-data control of the PVS of inverted pendulum due to the variation of its cart mass. The inverted pendulum is mounted on a motor-driven cart as shown in Fig. 4. In this case the pendulum moves only in x direction of the plane. The inverted pendulum problem is in fact an example of producing a stable closed-loop control system from an unstable plant. For this system, it is possible to design a controller using the pole placement techniques [16,17].

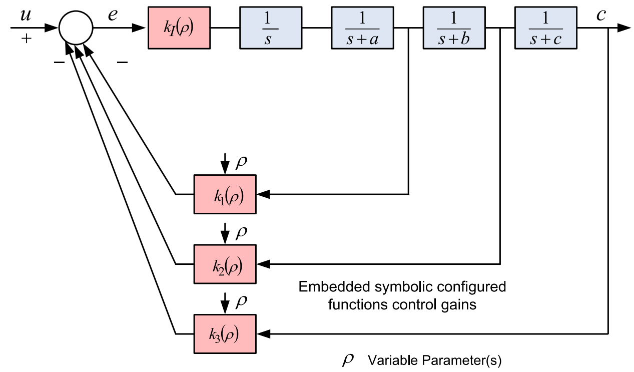


Fig. 3. Realization of inverted pendulum stabilization using proposed embedded symbolic-based configured expressions control gains derived through computational mathematics.

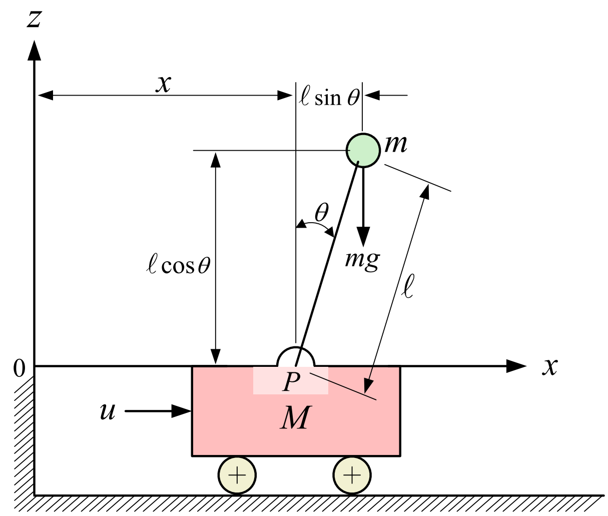


Fig. 4. The case study of continuous (infinite) modal M^{∞} system control and operation of the inverted pendulum mounted on a motor-driven cart.

The mathematical model for the case study can be expressed as follows:

$$M.l.\ddot{\theta} = g.\theta.(M+m) - u \quad \text{and} \quad M.\ddot{x} = u - m.g.\theta \tag{21}$$

where

x	The location of the pendulum cart.
u	The applied control force to the pendulum cart.
m	The mass of the pendulum ball.
M	The mass of the pendulum cart.
l	The length of pendulum rod.
θ	The angle of the pendulum rod from the vertical line.
g	The gravity of earth and the center of gravity is the center of the pendulum ball.

It is assumed that θ is small and the second-order terms $(\dot{\theta}^2)$ can be neglected, and then we can define the state variables of the inverted pendulum system as (g=9.81 g/m): $x_1 = \theta, x_2 = \dot{\theta}, x_3 = x, and\ x_4 = \dot{x}$. Accordingly, the general PVS mathematical modeling of the pendulum can be written in the form of state space representation as:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \tfrac{g.(M+m)}{M.l}.x_1 - \tfrac{1}{M.l}.u\\ \dot{x}_3 &= x_4,\\ &\text{and}\\ \dot{x}_4 &= \tfrac{-g.m}{M}.x_1 + \tfrac{1}{M}.\end{aligned} \tag{22}$$

Subsequently, the state-space matrix form of the inverted-pendulum system can be represented as follow:

$$\begin{aligned}\dot{x} &= A.x + B.u\\ &\text{and}\\ y &= C.x.\end{aligned} \tag{23}$$

Or in detailed symbolic matrix form respectively as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{g.(M+m)}{M.l} & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ \frac{-g.m}{M} & 0 & 0 & 0\end{bmatrix}.\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ -\frac{1}{M.l}\\ 0\\ \frac{1}{M}\end{bmatrix}.[u] \tag{24}$$

and

$$y = [0 \quad 0 \quad 1 \quad 0].\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}. \tag{25}$$

In this is study, the target is to keep the inverted pendulum in a vertical position by moving the cart. This requires the indication of cart position –signal (y) - taken back to the input and an integrator is inserted in the feed forward path, as shown in Fig. 5. The assigned closed-loop poles of the stabilized system are assumed as $\mu_1, \mu_2, \mu_3, \mu_{4,} and\ \mu_5$ (could be complex conjugates).

Similar approach for poles placement of Section 5 is applied for this case study. The MuPAD symbolic control solution obtained by the computational mathematics program for the exact generic stabilization of the parameter varying inverted pendulum can be expressed as [16,17]:

$$k_1 = \frac{M.l^2.\left[\frac{g.m.\sigma_1}{M} + \frac{g^3.m.(M+m)^2}{M^3.l^2} + \frac{g^2.m.(M+m).\sigma_2}{M^2.l}\right]}{g^2} - \frac{M.l^3.\left[\frac{g^3.(M+m)^3}{M^3.l^3} + \frac{g.(M+m).\sigma_1}{M.l} + \frac{g^2.(M+m)^2\sigma_4}{M^2.l^2}\right]}{g^2} + \frac{M.l.\left[\frac{g.m.\sigma_2}{M} + \frac{g^2.m.(M+m)}{M^2.l}\right]}{g} \tag{26}$$

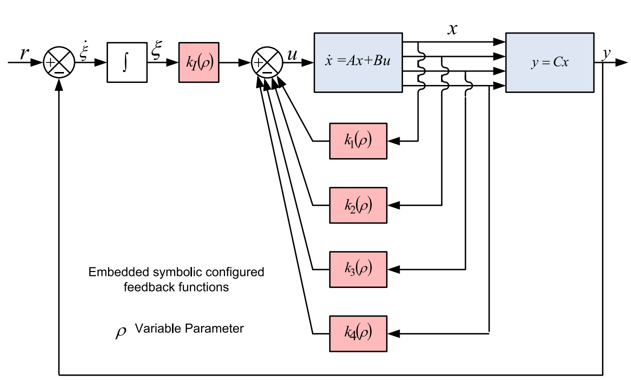


Fig. 5. Implementation of the inverted pendulum symbolic-based continuous (infinite) M^∞ modal scheme using embedded configured functions.

$$k_2 = \frac{M.l^3.\left[\sigma_5 + \frac{g^2.(M+m)^2.\sigma_3}{M^2.l^2} + \frac{g.(M+m).\sigma_4}{M.l}\right]}{g^2} - l.m.\sigma_3 - \frac{Ml^2.\left[\frac{g.m.\sigma_4}{M} + \frac{g^2.m.(M+m).\sigma_3}{M^2.l}\right]}{g^2} \tag{27}$$

$$k_3 = -\frac{M.l.\sigma_1}{g} \tag{28}$$

$$k_4 = \frac{M.l.\sigma_4}{g} + \frac{\sigma_5.M.l^2}{g^2} \tag{29}$$

and

$$k_I = -\frac{\sigma_5.M.l}{g} \tag{30}$$

such that

$$\begin{aligned}\sigma_1 &= \mu_1.\mu_2.\mu_3.\mu_4 + \mu_1.\mu_2.\mu_3.\mu_5 + \mu_1.\mu_3.\mu_4.\mu_5 + \mu_2.\mu_{3.}\mu_4.\mu_5\\ \sigma_2 &= \mu_1.(\mu_2+\mu_3+\mu_4+\mu_5) + \mu_2.(\mu_3+\mu_4+\mu_5) + \mu_3.(\mu_4+\mu_5)\\ \sigma_3 &= \mu_1+\mu_2+\mu_3+\mu_4+\mu_5\\ +\mu_4.\mu_5\ \sigma_4 &= (\mu_1+\mu_2).(\mu_3+\mu_4).\mu_5 + (\mu_1+\mu_2+\mu_5)\\ &.\mu_3.\mu_4 + (\mu_4+\mu_5).\mu_1.\mu_2 \text{ and } \sigma_5 = \mu_1.\mu_2.\mu_3.\mu_4.\mu_5.\end{aligned} \tag{31}$$

For a given stabilized inverted pendulum design poles $\mu_1, \mu_2, \cdots, and \mu_5$, the values of $\sigma_1, \sigma_2, \cdots, and\ \sigma_5$ can be calculated beforehand using (31) and substituting in (26) to (30)[3]. This will leave the control gains k_i as functions of the varying parameters $M, m, and\ l$. These parameters values can be monitored online and then sent to the embedded control function units to be fed continuously inside the exact symbolic control function gains.

It must be pointed out that, the calculation of the control gains of $k_1, k_2, \cdots, k_I$ using the derived generic exact symbolic expressions of (26) and (31) with the knowledge of system parameters and desired eigen-values involve again simple straightforward mathematical operation of types $(+ - \tilde{A} \cdot \times)$ following the proper sequence of calculations.

A exact generic continuous (infinite) modal system for this case study will mathematically be based on simultaneously manipulating the assigned eigenvalues $\mu_1, \mu_2, \mu_3, \mu_{4,} and\ \mu_5$ as a function of the varying parameter(s) denoted as "ρ". The variable parameter (s) for this case study the pendulum cart mass "M". Thus, we have the general formula corresponding to (18) and (19) as:

$$\mu_i = f_i(M), \text{suchthat } i = 1, 2, \ldots, 5. \tag{32}$$

The selection of the functions $f_i(M)$ will be investigated during the implementation and experimentation part of the inverted pendulum stabilization given in the following section. The changes pattern versus time of the pendulum car mass could take various forms of the linear or nonlinear types that could usually be expressed in some sort of algebraic expressions.

7.2. Implementation and experimentation of continuous (infinite) modal M^∞ control and operation expressions

The embedded control system configuration of the continuous (infinite) modal active control of the inverted pendulum was realized in this experimentation thru microcontrollers/Proteus simulation configuration [25] as shown in Fig. 6.

Proteus is a high performance, fast, accurate and flexible simulation and is easy to configure to simulate wide ranges of architec-

[3] It must be noted that the left hand side values of (31) are always real for any selected assigned conjugative design poles

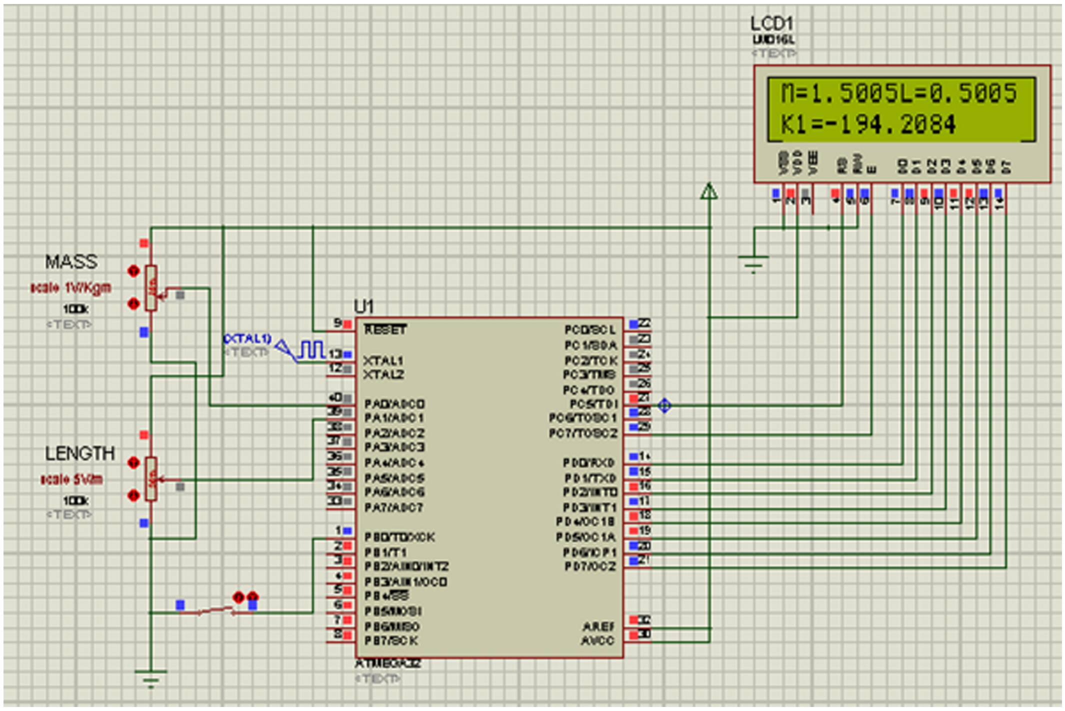


Fig. 6. Continuous (infinite) modal M^{∞} embedded control system implementation of the inverted pendulum using microcontroller.

tures. This arrangement was responsible for the control symbolic expressions executions (Functional Programming form).

The experiment was arranged to continuously reduce the cart mass of the inverted pendulum, and simultaneously manipulate the assigned system stabilization poles to cope with these changes based on a pre-selected mathematical (symbolic) formulations. The cart mass was monitored online and its sensed value is induced in the control symbolic expressions to create the corresponding values of control gains.

The experimental analysis of the inverted pendulum stabilization through symbolic-based control functions stabilization is carried out starting with the following initial points:

$$M_0 = 2.0\text{Kg}, m = 0.1\text{Kg}, l = 0.5\text{m}, g = 9.81\text{g/m}, \mu_1 = -1 + 3\text{j}, \mu_2 = -1 - 3\text{j}, \text{and } \mu_3 = \mu_4 = \mu_5 = -5.$$

This gives using (26) to (31), the initial point design for the control gains as:

$$k_1 = -157.6336, k_2 = -35.3733, k_3 = -56.0652, k_4 = -36.7466, \text{and } k_I = 50.9684.$$

In this experiment, the cart mas was continuously reduced according to the two following relationships scenarios:

(i) Scenario #1: Linear mass change versus time relationship

In this experiment, the cart mas was continuously reduced from M = 2.0 to 1.4 Kg within 100 seconds, following the linear relation: $M = M_0 - 0.006.t$.

(ii) Scenario #2: Nonlinear mass change versus time relationship

In this experiment, the cart mas was continuously reduced from M = 2.0 to 1.4 Kg within 100 seconds, following the nonlinear relation: $M = M_0 - 0.06.\sqrt{t}$

For both scenarios and at each value of monitored M, the corresponding assigned poles are simultaneously manipulated based on the following expressions:

$$\mu_1 = -1 + [(M - M_0)/0.3] + 3.j \text{ and} \\ \mu_2 = -1 + [(M - M_0)/0.3] - 3.j \tag{33}$$

and $\mu_3 = \mu_4 = \mu_5 = -5$ (are kept fixed during all continuous mode of operation).

The microcontroller (simulating the symbolic-based embedded configuration) then induced these updated poles continuously in the control gains expressions of (26) and (31) to calculate and implement the corresponding control gains. The results of monitored control gains of the experiments are plotted in Figs. 7 and 8 with the intermediate results of the experiments are given respectively in Tables 2 and 3.

Many other experiments were also successfully conducted by continuously online changing both the real and imaginary parts of the assigned poles with the change of the cart mass or by using different ranges of the assigned control parameters. All these experimental results had demonstrated the flexibility and effectiveness of implementing continuous (infinite) modal active control in real life application following the exact generic symbolic derivation approach. The embedded symbolic control configuration together with expression execution using computational mathematics have provided effective tools for smooth and complete compatibility of the proposed continuous (infinite) modal M^{∞} scheme.

Based on the above analysis, the scope of the new proposed symbolic-based approach versus corresponding numerical-based techniques reported in the literature for systems control and operation is shown in Fig. 9.

8. Applications to real life Cyber-physical systems

8.1. Cyber-Physical Symbolic-based control systems

The suggested embedded symbolic-based control system could be regarded as an evolutionary step towards the ***Cyber-physical***

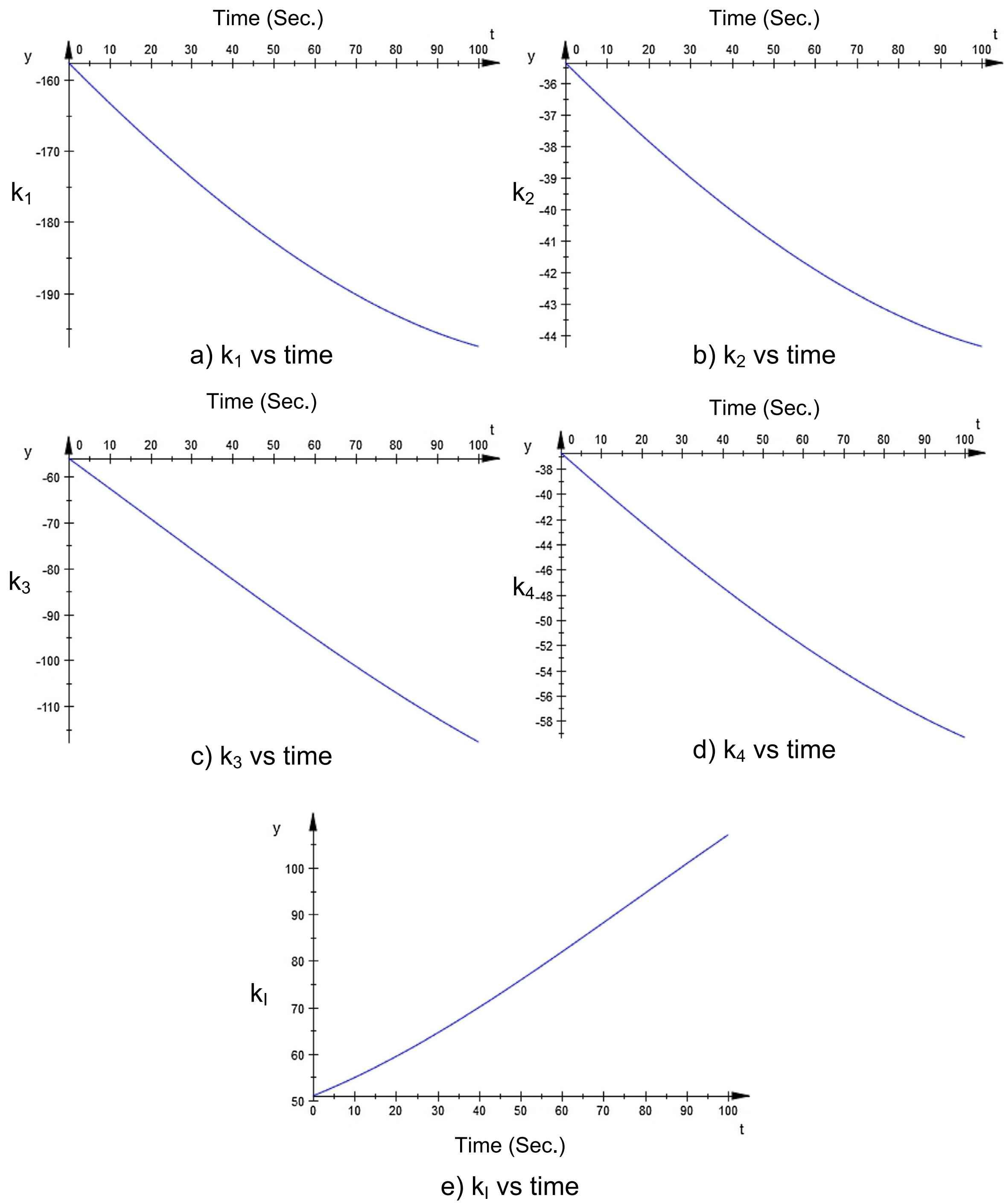


Fig. 7. Results of online changing of control gain corresponding to the continuous (infinite) modal M^{∞} cart mass **linear** changing behaviour versus time (***Scenario #1***).

direct control system as shown in Fig. 10**(a).** The system is affected by external environment causing continual parameter variation of the plant. The controller represents the "**Cyber**" part of the system, while the plant and its affecting environment represent the "**Physical**" part of system. Changes in the environment influencing the system parameters are monitored and transferred to the overall system through sensors and actuators [26].

An extended configuration of the above system is the ***Cyber-physical networked control system*** as elucidated in Fig. 10**(b).** In this extended case, a communication network is induced between the plant (physical entity) and the controller (computational entity). Such configuration, however, may experience induced delays in signals transmission, loss of transmitted packets (information) through the network, and possible effects of communication network on the system stability [27].

Thus, the configuration may not be suitable for the symbolic-based control of parameters varying systems of the fast-varying type. On the other hand, the extended configuration will permit reduced operation costs due to sharing the computational entity simultaneously within several applications, and the flexibility of placing the controllers in the adequate location not necessarily together or adjacent with the physical plant.

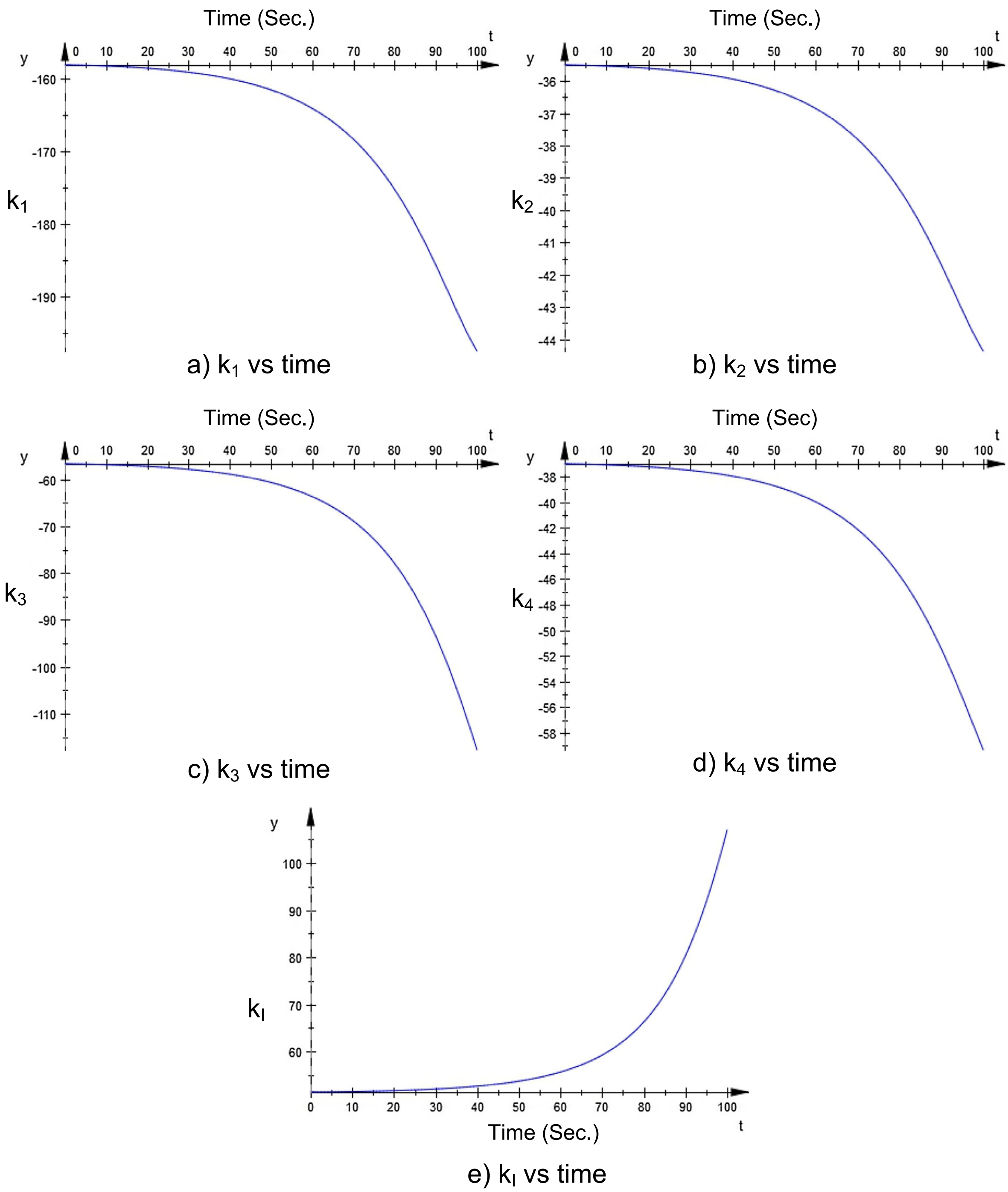


Fig. 8. Results of online changing of control gain corresponding to the continuous (infinite) modal M^∞ cart mass **nonlinear** changing behaviour versus time (***Scenario #2***).

Table 2
Summary of initial and intermediate experimental results corresponding to the continuous (infinite) modal M^∞ cart mass linear changing behaviour versus time (***scenario #1***).

Ser.	Parameter	Initial point	Intermediate experimental results					
1	Time (seconds)	0.0	16.667	33.333	50.000	66.667	83.333	100.0
2	M	2.0 kg	1.9 kg	1.8 kg	1.7 kg	1.6 kg	1.5 kg	1.4 kg
3	m	0.1 kg	0.1 kg	0.1 kg	0.1 kg	0.1 kg	0.1 kg	0.1 kg
4	Assigned changing $\mu's$	$-1.000+\sqrt{3}j$ $-1.000-\sqrt{3}j$	$-1.333+\sqrt{3}j$ $-1.333-\sqrt{3}j$	$-1.667+\sqrt{3}j$ $-1.667-\sqrt{3}j$	$-2.000+\sqrt{3}j$ $-2.000-\sqrt{3}j$	$-2.333+\sqrt{3}j$ $-2.333-\sqrt{3}j$	$-2.667+\sqrt{3}j$ $-2.667-\sqrt{3}j$	$-3.000+\sqrt{3}j$ $-3.000-\sqrt{3}j$
5	k_1	−157.63	−165.14	−174.97	−182.76	−188.06	−194.21	−197.48
6	k_2	−35.37	−37.06	−39.28	−41.04	−42.42	−43.65	−44.36
7	k_3	−56.06	−65.53	−77.67	−88.81	−99.05	−109.06	−117.73
8	k_4	−36.74	−40.69	−45.59	−49.78	−53.32	−56.70	−59.32
9	k_I	50.96	56.77	66.00	75.81	85.92	96.67	107.03

Table 3
Summary of initial and intermediate experimental results corresponding to the continuous (infinite) modal M^{∞} cart and ball masses nonlinear changing behaviour versus time (***scenario #2***).

Ser.	Parameter	Initial point	Intermediate experimental results					
1	Time (seconds)	0.0	16.667	33.333	50.000	66.667	83.333	100.0
2	M	2.0 kg	1.990 kg	1.980 kg	1.955 kg	1.901 kg	1.760 kg	1.4 kg
3	m	0.1 kg	0.0979 kg	0.0971 kg	0.0964 kg	0.0959 kg	0.0954 kg	0.0950 kg
4	Assigned changing μ's	$-1.000+\sqrt{3}.j$ $-1.000-\sqrt{3}.j$	$-1.031+\sqrt{3}.j$ $-1.031-\sqrt{3}.j$	$-1.064+\sqrt{3}.j$ $-1.064-\sqrt{3}.j$	$-1.141+\sqrt{3}.j$ $-1.141-\sqrt{3}.j$	$-1.328+\sqrt{3}.j$ $-1.328-\sqrt{3}.j$	$-1.797+\sqrt{3}.j$ $-1.797-\sqrt{3}.j$	$-3.000+\sqrt{3}.j$ $-3.000-\sqrt{3}.j$
5	k_1	−157.63	−158.53	−159.45	−161.62	−166.74	−178.33	−197.43
6	k_2	−35.37	−35.58	−35.79	−36.28	−37.44	−40.05	−44.364
7	k_3	−56.06	−57.101	−58.154	−60.669	−66.832	−82.265	−117.74
8	k_4	−36.74	−37.20	−37.65	−38.73	−41.30	−47.36	−59.33
9	k_I	50.96	51.55	52.15	53.66	57.73	69.90	107.03

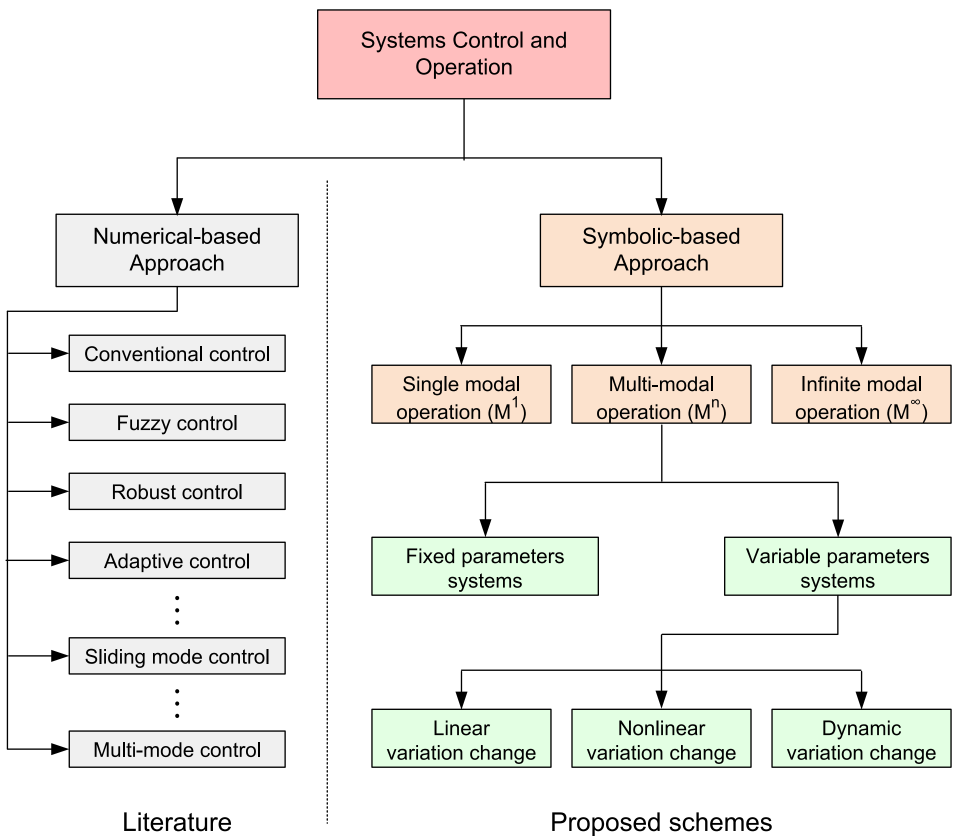


Fig. 9. A graph showing scope of the new proposed symbolic-based approach versus corresponding numerical-based techniques for systems control and operation.

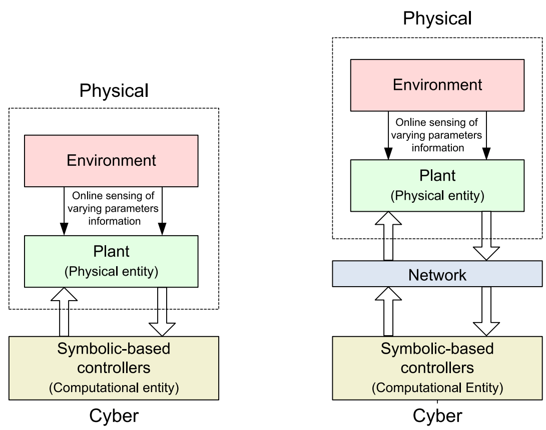
8.2. Example of control of for renewable energy generation systems

The main principle of symbolic-based control for renewable energy generation systems [28–30] could follow similar approach as above as shown in Fig. 11**(a,b)**. In Fig. 11**(a),** the symbolic-based controller is derived for the control of wind turbine energy generation where the online sensing of wind speed and wind direction information are fed to the exact generic symbolic-based controller tracking in an online form such changes. Similarly, same approach could be followed for solar systems control of Fig. 11 **(b)** where online sensing of solar radiation and temperature information are carried out and fed directly to adjust online the symbolic-based controller.

8.3. Cyber-Physical and human control system

There is a great resemblance and analogy between the Cyber-physical control system and the humans control system. Both systems have the same hierarchy of Fig. 10**(a)** operating under varying affecting environments. The humans control system is based on online sensing and perception processing mechanism that enables offering the appropriate control to the existing circumstances. There is a recent notion of **Cyber-physical and humans control system** where the humans' influences are induced within the control loop(s) of the overall system [31,32].

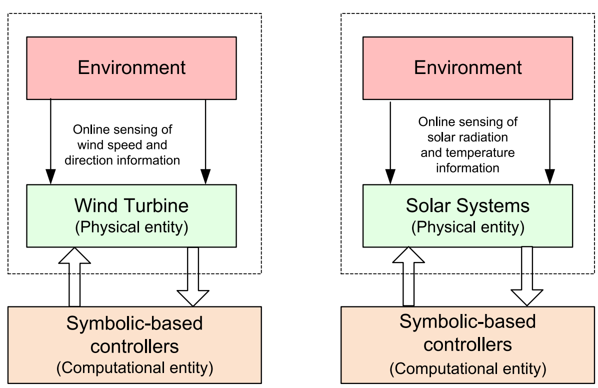
In essence, it is pointed out that the integration of symbolic-based components linked with relevant computational mathemat-



a) Cyber-physical direct symbolic-based control system

b) Cyber-physical networked symbolic-based control system

Fig. 10. Structure of Cyber-physical "**direct**" versus "**networked**" symbolic-based varying parameters control systems.



a) Cyber-physical symbolic-based control to wind turbine

b) Cyber-physical symbolic-based control to solar systems

Fig. 11. Cyber-physical symbolic-based direct control to renewable energy generation.

ical tools within physical systems and models in various disciplines is an important trend that must be strengthen. This trend could provide many control advantages and operation flexibilities compared to conventional systems and models [33–35].

9. Conclusions

The implementation of switching (finite) multimodal system control and operation is well known since the severities of the last century. In this case, depending on the mode of the varying basic system, operation parameters are selected corresponding to these changes. This arrangement, however, could cause non-smooth system behavior when moving from one mode to another.

In this paper, the transfer from the discrete, switching (finite) or multimodal system was made to the continuous (infinite) modal $\boldsymbol{M}^{\infty}$ behavior. Such process could be carried out through the derivation of the mathematical form of the system control and operation expressions and implemented thru symbolic-based embedded configuration using functional programming. It is speculated that such new type of control design could have very wide scope of PVS applications such as in the fields of ***mechatronics, robotics*** and ***industrial operations*** where smooth and flexible system behavior are to be maintained all over the modes of system operation. This makes the incorporation of the continuous (infinite) modal $\boldsymbol{M}^{\infty}$ scheme having more adaptable performance compared to the unimodal and multimodal approaches.

The "**proof of concept**" of the proposed continuous (infinite) modal $\boldsymbol{M}^{\infty}$ control scheme for PVS was successfully demonstrated by a case study of control and operation of inverted pendulum with cart and ball masses having linear and nonlinear changing behaviour with time. The symbolic-based representation of the control parameters developed through computational mathematics has enabled an exact generic platform for active control implementation. The system control and operation were experimented using micro-controllers/Proteus simulation configuration thru continuously manipulating simultaneously the control gains as a continuous function of the online linear and nonlinear varying parameters changes. It was shown that the exact generic tracking of changing parameters was effectively achieved in very smoothly and with complete continuous modal compatibility. The approach can also be extended to parameters varying **Cyber-physical Systems** such that the varying parameters are monitored online and continuously fed to the symbolic-control function to yield exact and generic flexible control scheme.

The shift from the switching (finite) multimodal operation to continuous (infinite) modal $\boldsymbol{M}^{\infty}$ operation could represent a new leap in engineering systems control and operation design, especially for remote, unattended and self-operated real-life applications. Such new framework is very generic and could have very wide scope of real-life applications. The approach maintains continuous smooth system behavior at various operations points (**auto or self-modality**) and gives a very high flexibility of control system operation with varying parameters. Finally, it is pointed out that the most appropriate determination of the smooth continuous modal operation transition relationships versus system varying parameters for each application represents an important challenging aspect that could add a new dimension for future engineering systems control and operation.

References

[1] Leigh JR. Control theory- A guided tour. Third Edition. London, UK: The Institution of Engineering and Technology (IET); 2012.
[2] Simon JD, Mitter SK. A theory of modal control. Inf Control 1968;18:316–53.
[3] Poh S, Baz A. Active control of flexible structure using a modal positive position feedback controller. Washington, DC: Mechanical Engineering Department, The Catholic University of America; 1990.
[4] Singh SP, Pruthi HS, Agarwal VP. Efficient modal control strategies for active control of vibrations. J Sound Vib 2003;262:563–657.
[5] Littleboy DM, Nichols NK. Modal coupling in linear control systems using robust eigenstructure assignment. In: Linear Algebra and its Applications. Elsevier Journal; 1998. p. 359–79.
[6] Hou L, Zong G, Wu YQ. Finite-time control for discrete-time switched systems with time delay. Int J Control Autom Syst 2012;10(4):855–60.
[7] Brait C. Linear parameter-varying and time-delay systems – Analysis, observation, filtering and control. Berlin Heidelberg: Springer-Verlag; 2015.
[8] Luo X. Data-driven predictive control for continuous-time linear parameter varying systems with application to wind turbine. Int J Control Autom Syst 2017;15(2):619–26.
[9] Wilcox ZD. Nonlinear control of linear parameter varying systems with applications to hypersonic vehicles, Ph. D. Dissertation, University of Florida, USA, 2010.
[10] Dorrah HT, Gabr WI, Elsayed MS. Generic symbolic parameters varying systems frameworks versus other techniques: Returning back to the roots", Alexandria Engineering Journal (AEJ, Elsevier), 2018, https://doi.org/10.1016/j.aej.2017.11.021, Vol. 57, Issue 4, pp. 3577–94, December 2018, (http://www.sciencedirect.com/science/article/pii/S1110016818301522).
[11] Dorrah HT, Gabr WI, Elsayed MS. Derivation of symbolic-based embedded feedback control stabilization with experimentation, Journal of Electrical Systems and Information Technology (JESIT, Elsevier), https://doi.org/10.1016/j.jesit.2018.02.003, Articles in Press, 2017; also Paper # S2.5, IEEE-ACCS/PEIT' 2017 Alexandria Egypt, 5-8 Nov. 2017, Vol. 5, pp. 40-47.
[12] Anisimov AA, Kotov DG, Tararykin SV, Tyutikov VV. Analysis of parametric sensitivity and structural optimization of modal control systems with state controllers. In: Journal of Computer and Systems Sciences International. Pleiades Publishing; 2011. p. 698–713.

[13] Wolfram S. Symbolic mathematical computations. Commun ACM April 1985;28(4):390–4.
[14] Lincoin L. Computational mathematics. New York, USA: Willford Press; 2016.
[15] Matlab. Symbolic Math Toolbox, User's Guide, Ver. 2016b. The MathWorks Inc., Natick, MA, USA, 2016.
[16] Burns RS. Advanced control engineering. Oxford, UK: Butterworth Heinemann, Reed Elsevier Group; 2001.
[17] Ogata K. Modern control engineering. Upper Saddle River, New Jersey: Pearson Education International Prentice Hall; 2010.
[18] Pasetti A. Software frameworks and embedded control systems. New York: Springer; 2003.
[19] Hristu-Varsakelis D, Levine WS. Systems handbook of networked and embedded control. 2nd ed. Boston, USA: Birkhauser; 2008.
[20] Aminifa A. Analysis design, and optimization of embedded control systems. Ph. D. Dissertation, No. 1746. Department of Computer and Information Science, Linköping University, Sweden, 2016.
[21] Muden R. ASIC and FPGA verification. A guide to components models. San Francisco, USA: Morgan Kaufmann Publisher (Elsevier); 2005.
[22] Michaelson G. An introduction to functional programming through Lambda Calculus. New York, USA: Dover Publication; 2011.
[23] Cohn JS. Computer algebra and symbolic computation – mathematical methods. Natick, MA, USA: A. K. Peters Ltd; 2003.
[24] Khalgui M, Hanisch HM. Reconfigurable embedded control systems: Applications for flexibility and agility. Hershey, Pennsylvania USA: IGI Global; 2011.
[25] Brewer EA, Dellarocas CN, Collbrook A, Weihl WE. Proteus: A high-performance parallel-architecture simulator Technical Report. Cambridge, MA, USA: Massachusetts Institute of Technology; 1991.
[26] Merro M, Lanotte R. A Calculus of Cyber-Physical Systems, Presentation, Department of Computer Science, University of Verona, Italy, 6 March 2017.
[27] Kim K-D, Kumar PR. Cyber-physical systems: a perspective at the centennial. Proc IEEE 2012;100:1287–308. Special Centennial Issue.
[28] Borri A, Di Benedetto MD, Pola G. Towards a unified theory for the control of CPS: A symbolic approach. ERCIM News 2014;97. Cyber-Physical Systems.
[29] Kekatos N, Forets M, Frehse G. Modeling the Wind Turbine Benchmark with PWA Hybrid Automata (Experience Report). ARCH17. 4th International Workshop on Applied Verification of Continuous and Hybrid Systems, Apr 2017, Pittsburgh, USA, Vol. 48, pp. 100-113, EPiC Series in Computing.
[30] Shiau J-K, Lee M-Y, Wei Y-C, Chen B-C. Circuit simulation for solar power maximum power point tracking with different buck-boost converter topologies. Energies 2014;7:5027–46. doi: https://doi.org/10.3390/en7085027.
[31] Munir S, Stankovic A, Liang CM, Lin S. Cyber physical system challenges for human-in-the-loop control, Presentation. USA: Dept. Computer Science, University of Virginia; 2013.
[32] Bajcsy R. Cyber-physical systems with humans in the loop, presentation. Electrical Engineering & Computer Sciences, University of California, Berkeley; 2014.
[33] Gabr WI, Dorrah HT, Abdel Magiud MZ. On the Consolidity-Inhibitors Theory of Change, Sylwan Journal, Volume 163, Issue 9, September 2019, pp. 134-147 (Paper Code: 1566637634_A3grd).
[34] Gabr WI, Dorrah HT, El-Gindy SA. Symbolic-based Optimal Operation of Flexible Reconfigurable Networks (FRNs) Using Movable/Changeable Resources, Sylwan Journal, Volume 163, Issue 9, September 2019, pp. 196-219 (Paper Code: 1566993814_72Bgo).
[35] Alippi C, Ozawa S. Chapter 12 - Computational Intelligence in the Time of Cyber-Physical Systems and the Internet of Things. In: Robert Kozma, Cesare Alippi, Yoonsuck Choe, Francesco Carlo Morabito, editors. Artificial Intelligence in the Age of Neural Networks and Brain Computing. Academic Press; 2019. p. 245–63.

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